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AN IMPROVED TECHNIQUE FOR ESTABLISHING RESISTANCE RATIOS

by

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LITHO IN U.S.A.

AN IMPROVED TECHNIQUE FOR ESTABLISHING RESISTANCE RATIOS

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ABSTRACT

A previously disclosed but seldom used application of four-terminal resistors permanently connected in series is discussed. Techniques for obtaining accurate ratios by four-terminal series, parallel, and series-parallel connections are analyzed.

THE IMPORTANCE OF RESISTANCE RATIOS

One of the most important functions of a dc standards laboratory is the establishment of resistance ratios. The intercomparison of resistors, voltages, and currents and the calibration of most dc measuring equipment depends on the accuracy of these ratios. For instance, the legal value of the ohm is maintained at the National Bureau of Standards by means of a group of Thomas-type one-ohm standard resistors. Other resistance values are referenced to one ohm by using resistance ratio techniques. Even the intercomparison of these standards depends on an accurate determination of a one-to-one ratio.

WHERE TO ESTABLISH RESISTANCE RATIOS

Resistance ratios do not involve the unit of resistance. They can be accurately determined by any laboratory with proper equipment and technique. Of even greater importance, if highest accuracy is desired, each laboratory can establish the ratios at the time and place they are needed in order to minimize the effects of time, temperature, shipment, etc.

TWO-TERMINAL BUILD-UP RESISTORS

Resistance ratios can be established by the use of build-up resistors. Build-up resistors can be connected in series, parallel, or series-parallel combinations to establish desired ratios. One accurate ratio technique using build-up resistors is the calibration of a ratio device by the use of r plus n nominally equal two-terminal resistors. If r nominally equal resistors are combined (connected in series, parallel, or series-parallel) in one arm of a bridge and n resistors of the same nominal value are combined in the other arm, and the resistors are rotated in a cyclic order until each resistor occupies each position once, the average of the r plus n readings is the reading that would be obtained if all the resistors were equal. The ratios that may be established by this technique are limited only by one's ability to find the proper build-up resistor combinations and perserverance in taking the necessary r plus n readings. The most used and useful ratios are 1:1 and 1:10. Usually the build-up resistors are used to calibrate a ratio bridge, and the calibrated bridge is used to transfer from one resistor value to another.

If two-terminal build-up resistors are used they must be high enough in value to avoid problems caused by connection and lead resistances. For highest accuracy, resistors that are at least 100 ohms are required, especially for parallel connections.

USING FOUR-TERMINAL BUILD-UP RESISTORS

The remainder of this paper discusses ratio techniques using four-terminal build-up resistors.

Hamon (1) has shown how four-terminal resistors can be connected in series, parallel, or series-parallel. The resistors are permanently connected in series, but ten of them may be connected in parallel, or nine of them may be connected in series-parallel. Other combinations are possible, but these are the most useful for ten to one and one hundred to one ratios. A two-terminal representation of the connections, and the connected resistance values are shown in Figure 1.

The advantage of this technique can be seen from the similarity between the formulae in Figure 1. For instance, if one knows the value of the ten resistors in the parallel connection, the the value of the same resistors in the series

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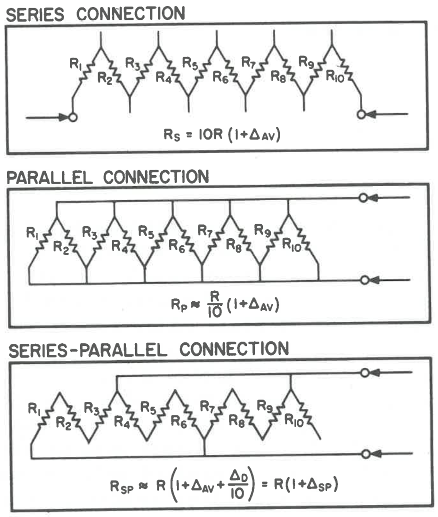
Numbers in parenthesis refer to similarly numbered references at the end of this paper.

connection is also known and one intercomparison, to find Δ_D, will give the value of the first nine resistors connected in series-parallel. These connections can be used for low resistance values too if similar four-terminal connections can be made accurately.


SERIES CONNECTION
R1 R2 R3 R4 R5 R6 R7 R8 R9 R10
$R_S = 10R(1+\Delta_{AV})$
PARALLEL CONNECTION
R1 R2 R3 R4 R5 R6 R7 R8 R9 R10
$R_P \approx \frac{R}{10}(1+\Delta_{AV})$
SERIES-PARALLEL CONNECTION
R1 R2 R3 R4 R5 R6 R7 R8 R9 R10
$R_{SP} \approx R\left(1+\Delta_{AV}+\frac{\Delta_D}{10}\right) = R(1+\Delta_{SP})$

Where R_i = nominal resistance of the ith resistor
Δ_i = deviation of R_i from R $\quad R_i = R(1+\Delta_i)$
R_S = resistance of 10 resistors in series
R_P = resistance of 10 resistors in parallel
R_{SP} = resistance of 9 resistors in series-parallel
Δ_{AV} = average deviation of the 10 resistors $\quad \Delta_{AV} = \frac{1}{10}\sum_{i=1}^{10}\Delta_i$
Δ_D = deviation of R_{SP} from the tenth resistor R_{10}
Δ_{SP} = deviation of R_{SP} from R $\quad \Delta_{SP} = \left(\Delta_{AV}+\frac{\Delta_D}{10}\right)$

These formulas are correct to about $\pm\left(\frac{1}{10}\sum_{i=1}^{10}\Delta_i^2 - \Delta_{AV}^2\right)$ (±1ppm if the build-up resistors are accurate to ±0.1% and there are no connection errors)

FIGURE 1. BUILD-UP RESISTOR CONFIGURATION

Figure 2 shows how three build-up resistor sets can be used to transfer from one ohm to all of the decade values up to one megohm using only one-to-one comparisons.

MEASUREMENT NUMBER	STANDARD RESISTOR	RESISTANCE LEVEL OF INTERCOMPARISON	UNKNOWN RESISTOR
1	1Ω (NBS certified)	1Ω	measure Δ_{AV} of ten 10Ω's in parallel
2	ten 10Ω's in series	100Ω	measure Δ_{AV} of ten 1k's in parallel
3	ten 1k's in series	10k	measure Δ_{AV} of ten 100k's in parallel
4	tenth (R_{10}) 10Ω	10Ω	measure Δ_D of nine 10Ω's in series-parallel, calculate Δ_{SP}
5	tenth (R_{10}) 1k	1k	measure Δ_D of nine 1k's in series-parallel, calculate Δ_{SP}
6	tenth (R_{10}) 100k	100k	measure Δ_D of nine 100k's in series-parallel, calculate Δ_{SP}

FIGURE 2. MEASUREMENTS (1:1 COMPARISONS) NECESSARY TO ESTABLISH STANDARDS OF RESISTANCE FOR THE SIX DECADE VALUES ABOVE 1Ω. CONNECTING TEN 100k'S IN SERIES GIVES THE 1 MEGOHM STANDARD

Some features of the resistance transfer technique described in Figure 2 are:

1. All of the required measurements are intercomparisons of nominally equal resistors that can be accomplished easily and accurately by a simple interchange or substitution. The bridge used must be stable during the time it takes to make one intercomparison and must have adequate resolution, sensitivity, and repeatability.
2. Only one intercomparison per decade of transfer is needed, and it is possible to transfer two decades per intercomparison if all of the decade values are not needed.
3. Some (measurements 4 through 6 in Figure 2) of the measurements are relatively non-critical. The measured deviation Δ_D is divided by ten when it is used.
4. The series-parallel standard can be used to calibrate a bridge for measurement of the individual resistors of itself. The series combinations of these resistors gives calibrated integer values for the calibration of non-decade valued resistors. To establish standards at integer values for all decades, build-up resistors of each decade value are required. The calibrated resistors can also be used as a voltage divider standard to calibrate Kelvin-Varley dividers, etc. (4)
5. The accuracy of the transfer technique depends on the short-term stability of the build-up resistors and does not require resistors of high long-term stability or known history.
6. If the build-up resistors do have good long-term stability, they can also be used for secondary standards of resistance and ratio in the time interval between calibrations.
7. The equipment used in the technique can be easily checked to determine the measurement accuracy.

ERROR REDUCTION

The remainder of this paper is an error analysis for four-terminal build-up resistor measurements.

The error sources to be considered are:

1. Resistor stability with changes in:
   a. Temperature
   b. Power
   c. Voltage
   d. Time
2. Junction, lead, and contact resistance effects

RESISTOR STABILITY

The time stability and ambient temperature coefficient for a build-up resistor are intimately connected. Both sources of error may be reduced by making a transfer in a short period of time. The average temperature of the build-up resistors is relatively unimportant, but changes in temperature during the transfer must be small to avoid significant changes in the resistor values. In practice, it is possible to measure the series, parallel, and series-parallel combinations in a time period of a few minutes. If commercially available low temperature coefficient resistors are mounted in a thermally lagged box, the short amount of time necessary to accomplish the measurements will keep the errors caused by temperature changes well below one ppm in a typical laboratory environment.

If wire-wound resistors are used for the build-up resistors, there will be no observable errors caused by voltage coefficient of the resistors.

The errors introduced by the power coefficients of the resistors may be reduced in two ways. One method is to keep the resistors at the same power level in all of the build-up configurations. The other is to operate the resistors at a low power level to avoid significant self-heating. The low power method is usually the most practical for the ten resistor technique. The power is always distributed between ten resistors in the series and parallel connections and nine resistors in the series-parallel connection. The only time a single resistor is used the measured deviation (Δ_p) is divided by ten (see Figure 2).

The power level which is used to avoid any short-term measurable resistance change resulting from self-heating may be higher than the power level which will result in a long-term observable resistance change. Since a resistor takes some time to heat after power is applied, the higher power level may be used if the resistors are measured in a short enough period of time.

FOUR-TERMINAL BUILD-UP RESISTORS IN SERIES

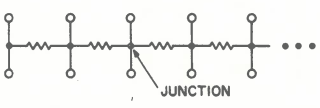


FIGURE 3. FOUR-TERMINAL BUILD-UP RESISTORS PERMANENTLY CONNECTED IN SERIES

The series configuration of the build-up resistors is shown in Figure 3. The four-terminal resistors can be measured individually or in any series combination. If the measured series resistance is to be equal to the sum of the individual resistor measurements, the junctions must have zero four-terminal resistance. The junction resistance is zero if a current source connected to any two of the four junction terminals produces zero voltage difference between the other two terminals.

In practice, the junctions do have some resistance. It is possible to make the resistance very low (much less than 1 microhm) by proper design. For instance, if four terminals are connected to a plane sheet of metal in the configuration shown in Figure 4, two of the terminals will always be on an equipotential line if a current source is connected to the other two terminals.

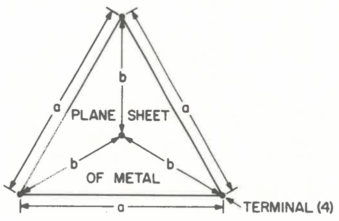


FIGURE 4. A FOUR-TERMINAL JUNCTION DESIGN

If the junction material has zero voltage coefficient and does not rectify current, a reasonable assumption for metals, Series (2) has shown that the junction can be represented by the equivalent circuit shown in Figure 5. It is not convenient to measure the resistors shown in this equivalent circuit. A set of resistances that can be measured are shown in Figure 6. The resistors shown in Figure 5 and 6 are related in the following manner.

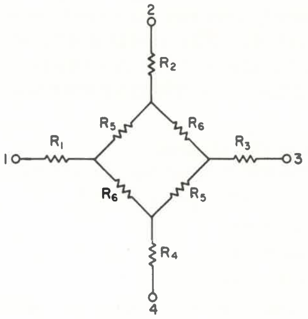


FIGURE 5. EQUIVALENT CIRCUIT OF FOUR-TERMINAL JUNCTION

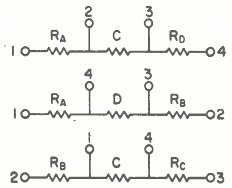


FIGURE 6. RESISTANCE MEASUREMENTS OF A FOUR-TERMINAL JUNCTION

$$R_1 = R_A - \sqrt{CD} \qquad R_A = R_1 + \frac{R_5 R_6}{2(R_5 + R_6)}$$

$$R_2 = R_B - \sqrt{CD} \qquad R_B = R_2 + \frac{R_5 R_6}{2(R_5 + R_6)}$$

$$R_3 = R_C - \sqrt{CD} \qquad R_C = R_3 + \frac{R_5 R_6}{2(R_5 + R_6)}$$

$$R_4 = R_D - \sqrt{CD} \qquad R_D = R_4 + \frac{R_5 R_6}{2(R_5 + R_6)}$$

$$R_5 = 2\left(D + \sqrt{CD}\right) \qquad C = \frac{R_6^2}{2(R_5 + R_6)}$$

$$R_6 = 2\left(C + \sqrt{CD}\right) \qquad D = \frac{R_5^2}{2(R_5 + R_6)}$$

Figure 7 shows the use of the equivalent circuit for the junction error when resistors are connected in series. If resistance R_{1234} is the notation for a four-terminal measurement defined by the ratio of the voltage drop from terminal 2 to terminal 3 to the current in terminal 1 and out terminal 4. The series error is the difference between the series measurement and the sum of the individual measurements.

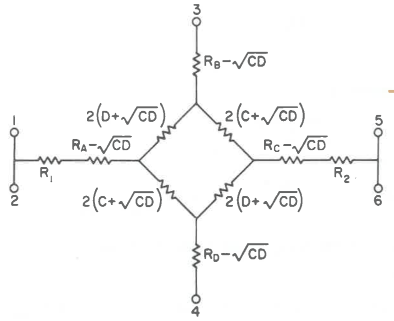


FIGURE 7. TWO FOUR-TERMINAL RESISTORS IN SERIES USING THE EQUIVALENT CIRCUIT FOR THE JUNCTION

SERIES ERROR FOR TWO RESISTORS

$$\text{Series resistance error} \equiv R_{2156} - R_{2134} - R_{4356} = C + D$$

$$R_{2156} = \left(R_1 + R_A\right) + \left(C + D\right) + \left(R_2 + R_C\right)$$

$$R_{2134} = R_1 + R_A$$

$$R_{4356} = R_2 + R_C$$

SERIES ERROR FOR n RESISTORS

If C and D are less than ±M, $(-M<C<+M \text{ and } -M<D<+M)$

For all junctions and $R_1 + R_A \approx R_2 + R_C \approx R$

Then $-2M<(C+D)<+2M$

And series error $< \pm \frac{2M}{R} \times \frac{n-1}{n}$ in proportional parts

FOUR-TERMINAL BUILD-UP RESISTORS IN PARALLEL

There are two sources of errors to be considered when the four-terminal build-up resistors are connected in parallel and series-parallel. These are the errors arising from the junctions which connect the build-up resistors in series, and the errors arising from resistances of the leads and connections which complete the parallel circuit.

The circuit used in evaluating the junction errors is shown in Figure 8. The upper junction is evaluated assuming that the lower junction is perfect. The parallel error is the difference between the measured parallel resistance and the parallel resistance computed from measurements of the individual resistors.

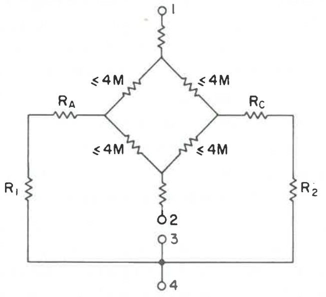


FIGURE 8. FOUR-TERMINAL RESISTORS IN PARALLEL USING THE EQUIVALENT CIRCUIT JUNCTION

PARALLEL JUNCTION ERROR

$$-M \leqslant \left[R_{1234} - \frac{\left(R_1 + R_A\right)\left(R_2 + R_C\right)}{R_1 + R_A + R_2 + R_C}\right] \leqslant \pm M$$

M = limit of junction resistance $|C|$ and $|D| < M$

R_1 and $R_2 \approx R$

R_A, R_C, C and D << R

Figure 9 shows the equivalent circuit of two four-terminal resistors connected in parallel. R_1 and R_2 are the resistors to be measured and $R_3, R_4, \cdots R_{10}$ represent lead and connection resistances. Wenner (3) has shown that the measured resistance R_{1234} will be equal to the calculated parallel value of R_1 and R_2 if:

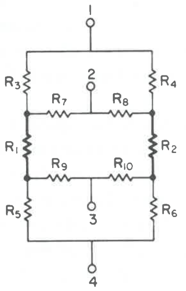


FIGURE 9. CIRCUIT USED TO CONSIDER LEAD AND CONNECTION ERRORS FOR TWO FOUR-TERMINAL RESISTORS IN PARALLEL

Either

$$\frac{R_1}{R_5} = \frac{R_2}{R_6} \quad \text{and} \quad \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

Or

$$\frac{R_1}{R_7} = \frac{R_2}{R_8} \quad \text{and} \quad \frac{R_1}{R_9} = \frac{R_2}{R_{10}}$$

Or if

$$R_3 = R_4 = R_5 = R_6 = 0 \quad \text{or} \quad R_7 = R_8 = R_9 = R_{10} = 0$$

If these conditions are not met completely further analysis of the parallel connection of the resistors will yield the error Δ_P for the measured parallel resistance R_P in terms of the maximum ratio unbalance Δ_M and the maximum bus bar to build-up resistance ratio Q.

PARALLEL CONNECTION ERROR

$$R_P = \frac{R_1 R_2}{R_1 + R_2}\left(1 + \Delta_P\right)$$

Where Δ_P is:

$$\Delta_P < Q\,\Delta_M$$

When Δ_M is:

$$\Delta_1\,,\ \Delta_2 < \Delta_M$$

For Δ_1, Δ_2 of:

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}\left(1+\Delta_1\right) \qquad \frac{R_1}{R_5} = \frac{R_2}{R_6}\left(1+\Delta_2\right)$$

And Q is:

$$\frac{R_7 + R_8}{R_N}\,,\ \frac{R_9 + R_{10}}{R_N} < Q$$

For

$$R_1 \approx R_2 \approx R_N$$

In order to keep the error Δ_P low, both Q and Δ_M must be small. Q can be made small by reducing the resistances R_7+R_8 and R_9+R_{10}. Δ_M can be thought of as the maximum unbalance of two bridges. One is made up of the resistors R_1, R_2, R_3, and R_4 and the other bridge is made up of the resistors R_1, R_2, R_5, and R_6. Δ_M can result from lack of precision matching of the resistances $R_1, R_2, \cdots R_6$ or from uncertainty in the contact resistances when R_3, R_4, R_5, and R_6 are connected. The latter source of error can be reduced by making R_3, R_4, R_5, and R_6 large compared to the uncertainties in the contact resistances.

TEN FOUR-TERMINAL BUILD-UP RESISTORS IN PARALLEL

The parallel connection of the ten four-terminal build-up resistors is shown in Figure 10.

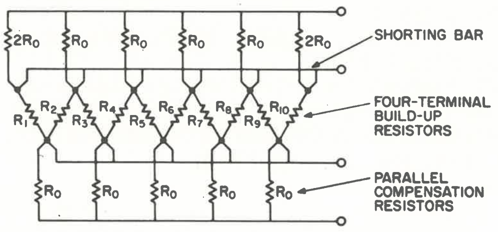

2Ro
Ro
Ro
Ro
Ro
2Ro
SHORTING BAR
R1
R2
R3
R4
R5
R6
R7
R8
R9
R10
FOUR-TERMINAL BUILD-UP RESISTORS
Ro
Ro
Ro
Ro
Ro
PARALLEL COMPENSATION RESISTORS

FIGURE 10. TEN FOUR-TERMINAL BUILD-UP RESISTORS IN PARALLEL

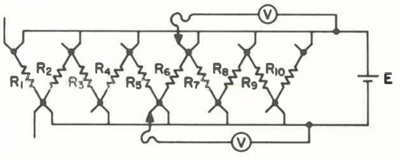
The shorting bars and the resistance from the junctions to the shorting bars are equivalent to the resistances R_7, R_8, R_9, R_{10} in Figure 9. These resistances should be small. The parallel-compensation resistors are equivalent to the resistances R_3, R_4, R_5, and R_6 in Figure 9. These resistances should be large compared to the uncertainty in the contact resistances when they are connected and should be proportional to the resistance of the build-up resistors. The two parallel-compensation resistors on the ends are double the others because they connect to one build-up resistor instead of two in parallel.

When several nominally equal resistors are connected in parallel the connection accuracy can be analyzed by assuming all of the compensation resistors except one are equal, and that all of the bus bar resistances except one are zero. At the same time all of the resistors being connected are assumed to be perfect except one. The three imperfect resistors all meet at one of the junctions. Since small first order error effects add linearly the results of this analysis can be extended to determine the connection accuracy. The connection uncertainty is less than:

$$\pm 2\left(\frac{r}{R}\right)\left(\Delta_{R_0} - \Delta_R\right)$$

Where
r = greatest bus bar resistance
R = nominal value of resistors being parallel connected
$\left(\Delta_{R_0} - \Delta_R\right)$ = greatest bridge unbalance in terms of resistance deviations of the compensation resistors and the resistors being connected

The values of $\frac{r}{R}$ and $\left(\Delta_{R_0}-\Delta_R\right)$ can be measured to find the expected accuracy of a particular connection.



Connect the shorting bar

Measure voltage from a point on the bars to the unused terminals, this is the voltage to the junction of adjacent resistors

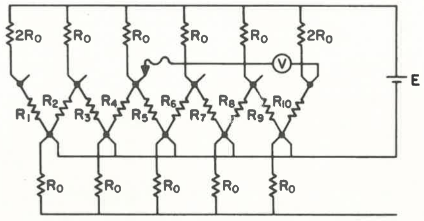
Find the maximum difference of values V for each bar

Use the greatest V difference to calculate $\left(\frac{r}{R}\right)_{MAX}$

$$\text{for } r \ll R \qquad \frac{V_{MAX} - V_{MIN}}{E} \approx \left(\frac{r}{R}\right)_{MAX}$$

FIGURE 11. MEASURING THE SHORTING BAR RESISTANCE r

The value of $\frac{r}{R}$ can be found by measuring the voltage drops from a point on the bus bars to the junction of adjacent resistors, shown in Figure 11.



Connect the compensating network

Connect one shorting bar

Supply a voltage E from the shorting bar to the other side of the network

Measure the voltage V from one unused shorting bar terminal to each of the others

Change the shorting bar to the other side and repeat

Find the maximum voltage difference $\left(V_{MAX} - V_{MIN}\right)$ between terminals

Calculate $\left(\Delta_{R_0}-\Delta_R\right)_{MAX}$

$$\left(\frac{V_{MAX} - V_{MIN}}{E}\right)\left(2 + \frac{2R_0}{R} + \frac{R}{R_0}\right) = \left(\Delta_{R_0}-\Delta_R\right)_{MAX}$$

FIGURE 12. MEASURING THE 'BRIDGE UNBALANCE' $\left(\Delta_{R_0} - \Delta_R\right)$

The bridge unbalance can be measured as shown in Figure 12. The measured values of $\left(\frac{r}{R}\right)_{MAX}$ and $(\Delta_{R_0}-\Delta_{R})_{MAX}$ can be multiplied together and doubled to give an upper bound of the connection error. This usually includes a very substantial safety factor. The same technique can be used in the series-parallel case.

MEASURING FOUR-TERMINAL BUILD-UP RESISTORS

The value for the parallel compensation resistors is a compromise. If they are too small the connection resistance will cause too much variation. If they are too large they will complicate the four-terminal measurement. For instance, if a Kelvin Bridge is used for the measurement, some of the parallel compensation resistors will be in series with the yoke of the bridge. If the yoke resistance is high relative to the measured resistance the measurement must be done carefully. There are at least two ways to reduce any problems that might arise. One method requires a more precise balance of the inner arms of the bridge, relative to the outer arms. Another method is to use a technique, such as interchange, which keeps the yoke resistance constant.

CONCLUSION

Consideration of the results shown above will show that it is possible to make four-terminal build-up resistor equipment that will permit the connection of 1Ω and higher valued resistors in series, parallel, or series-parallel with 1 ppm precision without the use of amalgamated mercury connections. The authors have assisted in the development of equipment of this type, some of which is shown in Figure 13. Figure 14 shows a practical junction design. All of the four-terminal resistances of these junctions are less than 0.5 microhms and can be made smaller if necessary. The measurement accuracy of this equipment can be predicted by additional measurements to show its connection accuracy.

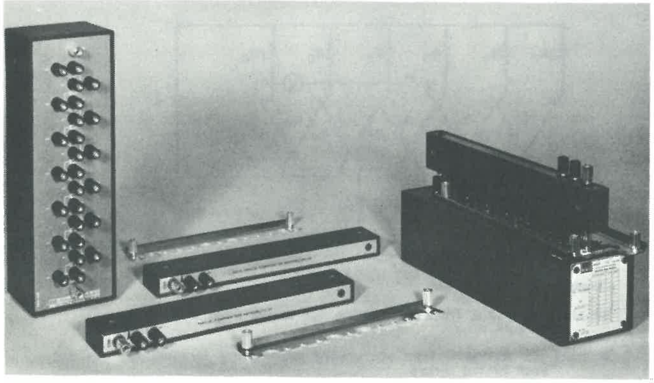


FIGURE 13. SOME FOUR-TERMINAL BUILD-UP RESISTOR EQUIPMENT

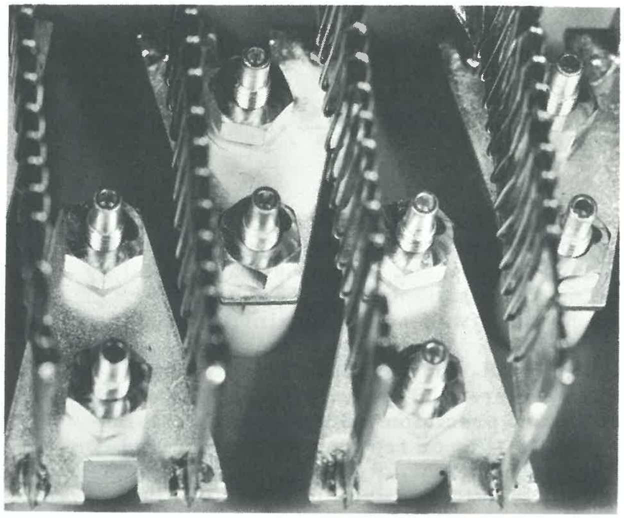


FIGURE 14. FOUR-TERMINAL JUNCTIONS

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